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MATHEMATICAL MODELING OF LIQUID FLOW BY THE FINITE ELEMENT METHOD

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Liquids are one of the most important natural phenomena that humanity faces and human life depends on. Watching the movement of water in a river, lake or ocean, we are amazed at the beauty and naturalness of moving water, its mobility and flexibility. After all, it seems that there is nothing simpler than water, which is used and without which there is no life.

Paradoxical, but true. We know next to nothing about how water behaves. The resulting mathematical models were difficult to solve, and the results of the analysis were surprising. In the end, this leads us to believe that we may have missed something underlying the movement of the fluid along this path.

The equations of a viscous incompressible fluid with density ρ and viscosity v on the plane $u(x,t) = (u, v, w)$ and pressure p have the form below (Navier-Stokes equation):

$$\frac{\partial u}{\partial t} + (u\nabla)u = -\frac{1}{\rho}\nabla p + v\Delta u - F, \quad (1)$$

$$\nabla \cdot u = 0. \quad (2)$$

Here x is the Euler coordinates, t is the time, F is the body force. According to the type of determination of the boundary conditions at the boundary of the wave Ω occupied by the liquid, two main installations are distinguished: a) with a velocity vector specified in Γ; b) a stream of written problems; for example, the pressure and contact velocity components are shown on the Γ_1 (inlet) and Γ_2 (outlet) boundaries, and the velocity vector is shown on the rest of the boundary. A 2D plot is plotted with wall equation $u = 0$, inlet equation $u =- U_0 n$, outlet equation

$$\begin{cases}[-pl + K]n = -\widehat{p_0}n \\ \widehat{p_0} \leq p_0\end{cases},$$

and pressure $p_0 = 0\ Pa$.

For (1)-(2), obtaining analytical solutions to boundary value problems is impossible due to non-linearity. The study of the properties of solutions is usually carried out by solving a series of simplified mathematical equations. The use of the finite element method provides a detailed and accurate representation of the physical processes occurring in these systems. Figure 1 shows that the flow velocity on the walls, obtained at a flow rate of 0.001 m/s, is greater than that of water flowing at a speed of 0.01 m/s, shown in figure 2. The flow of water can be seen with horizontal lines.

The finite element method involves dividing the physical space into small interconnected elements, each of which is represented by a set of nodes (figure 3 and figure 4). In these nodes, the values of physical properties such as flow rate and pressure are calculated. The values at the nodes are then used to generate a global solution for the entire system [1]. The mesh has 6381 triangles, 582 quads and a total of 6963 elements and 3939 nodes.

One of the main advantages of using the finite element method when modeling groundwater systems is the ability to work with complex geometries. This method can be applied to both simple and complex systems such as multilayer soil and rock layers, and can also be used to model systems with highly variable properties such as permeability and porosity [2].

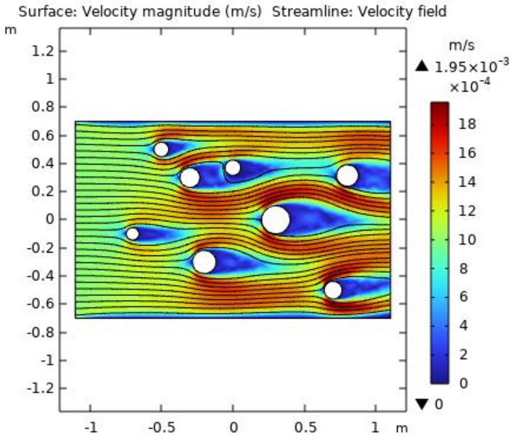

Figure 1. 2D plot with horizontal lines with flow velocity U_0=0.001 m/s

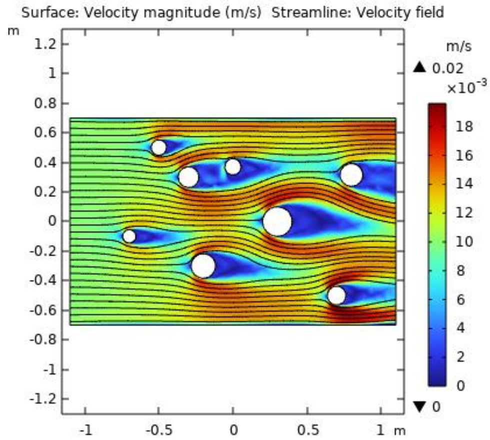

Figure 2. 2D plot with horizontal lines with flow velocity U_0=0.01 m/s

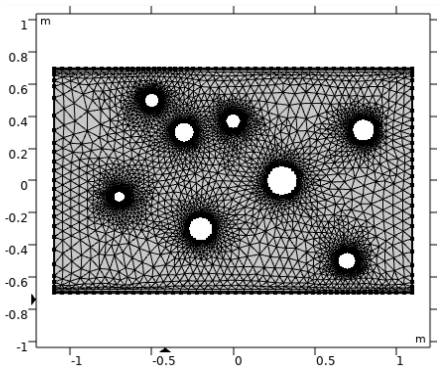

Figure 3. Mesh distribution of water flow

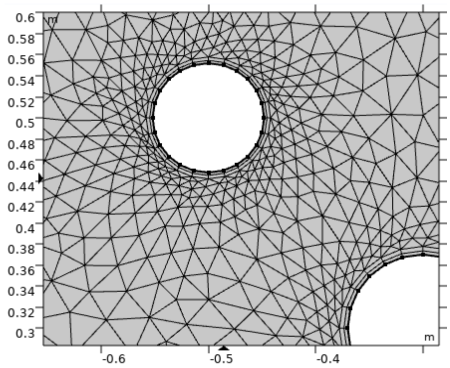

Figure 4. Enlarged section of the mesh distribution of water flow

In addition, the finite element method makes it possible to take into account various boundary conditions, such as the presence of wells or rivers, which play an important role in the behavior of groundwater systems. This method also allows the simulation of time-dependent processes such as groundwater recharge and depletion.

However, the finite element method also has some limitations. One of them is the high computational cost associated with solving large systems of equations, which can be a major problem when dealing with large and complex systems. In addition, the accuracy of the solution can be affected by the quality of the grid used to divide the area into elements. Therefore, it is important to select the appropriate mesh size and element type to obtain accurate results.

Therefore, the finite element method is a powerful tool for the mathematical modeling of groundwater systems. Decomposing a system into small interconnected elements provides a detailed and accurate representation of the physical processes occurring in the system. But this is computationally intensive and it is important to choose the appropriate mesh size and element type to get accurate results.

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